Section 3.7
Implicit Differentiation

$y = x^2$ EXPLICIT

$xy + y^2 = 5$ IMPLICIT - "Hidden"
y is a function of x

Example

Differentiate

$y^2 = x$

Differentiate both sides with respect to x
Then solve for $\frac{dy}{dx}$

$$\frac{2y\frac{dy}{dx}}{2y} = \frac{1}{2y}$$

$$\frac{dy}{dx} = \frac{1}{2y}$$

$x^3 + y^3 - 9xy = 0$

$$3x^2 + 3y^2\frac{dy}{dx} - 9\left((1)(y) + (x)(1)\frac{dy}{dx}\right) = 0$$

$$3x^2 + 3y^2\frac{dy}{dx} - 9y - 9x\frac{dy}{dx} = 0$$

$$3y^2\frac{dy}{dx} - 9x\frac{dy}{dx} = -3x^2 + 9y$$

$$\frac{dy}{dx}(3y^2 - 9x) = -3x^2 + 9y$$

$$\frac{dy}{dx} = \frac{-3x^2+9y}{3y^2-9x} = \frac{\cancel{3}(-x^2+3y)}{\cancel{3}(y^2-3x)} = \frac{-x^2+3y}{y^2-3x}$$

$\cos^2 y - \underbrace{\sqrt{x^3 + 1}}_{(x^3+1)^{1/2}} = 5 + x$

$$2\cos y \cdot (-\sin y)\frac{dy}{dx} - \frac{1}{2}(x^3+1)^{-\frac{1}{2}}(3x^2) = 1$$

$$-2\cos y \sin y\frac{dy}{dx} = 1 + \frac{1}{2}(x^3+1)^{-\frac{1}{2}}(3x^2)$$

$$\frac{dy}{dx} = \frac{1 + \frac{1}{2}(x^3+1)^{-\frac{1}{2}}(3x^2)}{-2\cos y \sin y}$$

$$= \frac{1 + \frac{3x^2}{2\sqrt{x^3+1}}}{-2\cos y \sin y}$$

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Find $\frac{d^2y}{dx^2}$ if $2x^3 - 3y^2 = 8$

Find the slope of the circle $x^2 + y^2 = 25$ at the point (3, -4)

Find the tangent and normal to the ellipse $x^2 - xy + y^2 = 7$ at the point (-1, 2).